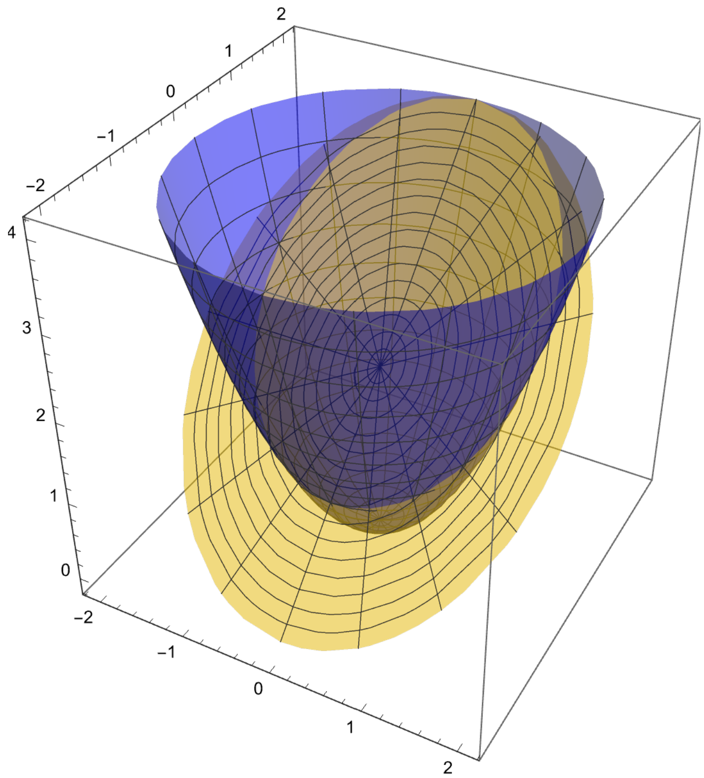
Plane and paraboloid problem from Wednesday, March 27th

In[451]:=

```
ParametricPlot3D[{{r Cos[θ], r Sin[θ], r^2}, {r Cos[θ], r Sin[θ], r Sin[θ] + 2}},
 {r, 0, 2}, {θ, 0, 2 π}, PlotStyle → {{Blue, Opacity[0.5]}, {Yellow, Opacity[0.5]}}]
```

Out[451]=

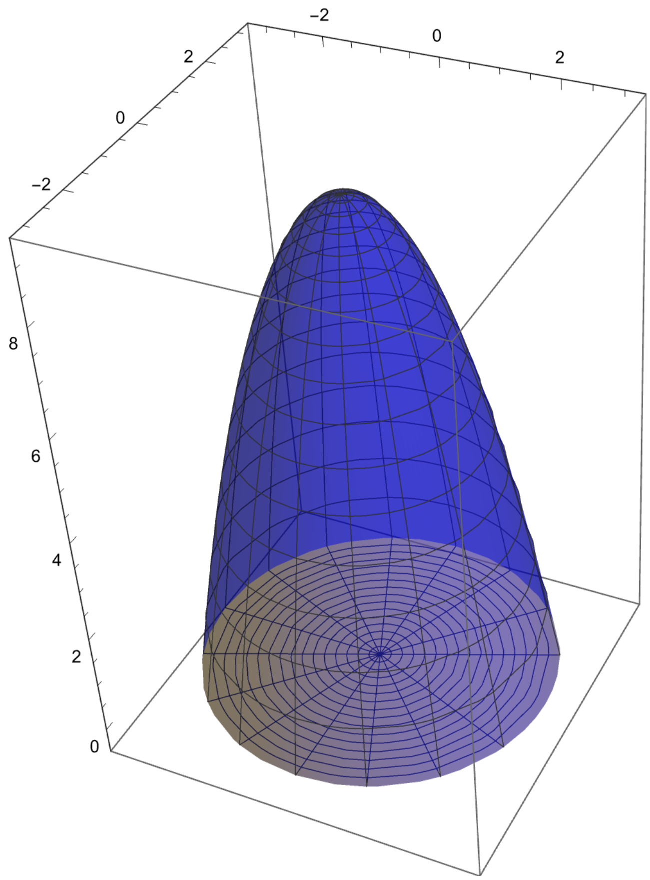


Paraboloid and xy-plane center of mass problem from 4/1:

In[452]:=

```
ParametricPlot3D[{{r Cos[θ], r Sin[θ], 9 - r^2}, {r Cos[θ], r Sin[θ], 0}}, {r, 0, 3},
 {θ, 0, 2 π}, PlotStyle → {{Blue, Opacity[0.5]}, {Yellow, Opacity[0.5]}}]
```

Out[452]=



volume of solid using cylindrical:

In[456]:=

$$\text{vol} = \int_0^{2\pi}\left(\int_0^3\left(\int_0^{9-r^2} 1\, r\, dz\right) dr\right) d\theta$$

Out[456]=

$$\frac{81\,\pi}{2}$$

first moment using cylindrical:

In[457]:=

$$\text{Mxy} = \int_0^{2\pi}\left(\int_0^3\left(\int_0^{9-r^2} z\, r\, dz\right) dr\right) d\theta$$

Out[457]=

$$\frac{243\,\pi}{2}$$

which gives zbar, the z-coordinate of the center of mass as

In[458]:=

$$\text{zbar} = \frac{\text{Mxy}}{\text{vol}}$$

Out[458]=

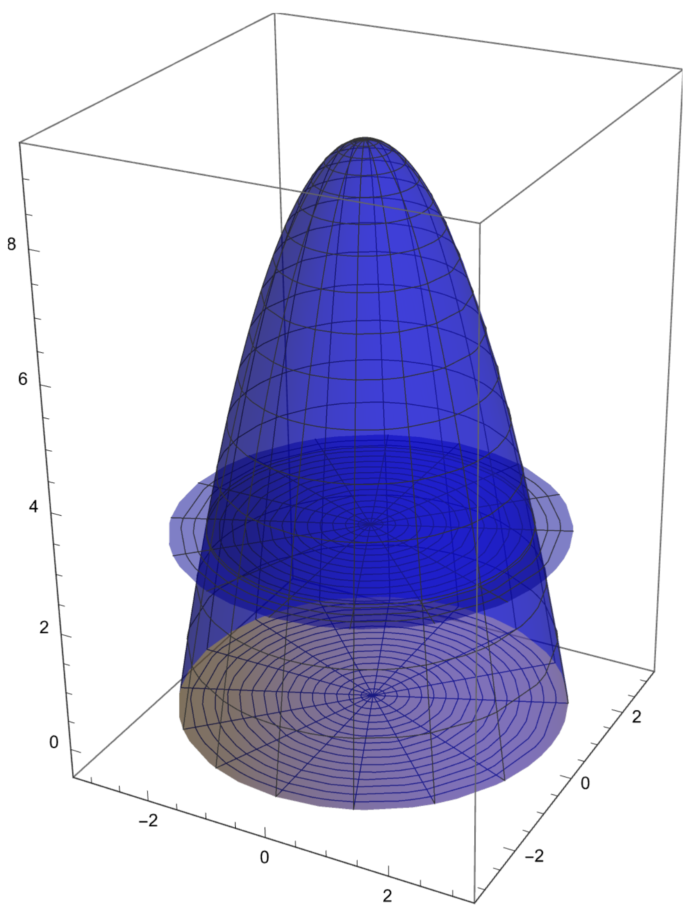
3

which indeed is less than halfway up the solid, which included z-values with range 0 to 9:

In[464]:=

```
ParametricPlot3D[
 {{r Cos[θ], r Sin[θ], 9 - r^2}, {r Cos[θ], r Sin[θ], 0}, {r Cos[θ], r Sin[θ], 3}},
 {r, 0, 3}, {θ, 0, 2 π}, PlotStyle → {{Blue, Opacity[0.5]}, {Yellow, Opacity[0.5]}}]
```

Out[464]=



The second integral, to find the first moment across the xy-plane, in smaller steps:

In[465]:=

$$\int_0^{9-r^2} z\, r\, dz$$

Out[465]=

$$\frac{1}{2} r \left(9 - r^2\right)^2$$

In[466]:=

$$\int_0^3 \left(\int_0^{9-r^2} z\, r\, dz\right) dr$$

Out[466]=

$$\frac{243}{4}$$